Calculus Warm Up # 1- 5

Find the limits:

1. $\lim_{x \to 1} \dfrac{x^3 - 1}{x - 1}$

2. $\lim_{x \to 2} \dfrac{x^3 - 2x^2 + 2x - 4}{x - 2}$

3. $\lim_{x \to 7^+} \dfrac{x - 7}{|x - 7|}$

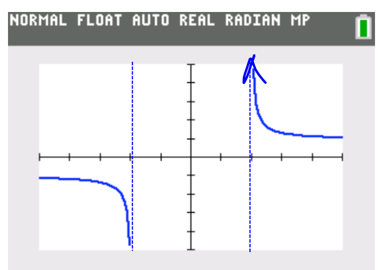
4. $\lim_{x \to 2^-} [\![3x - 1]\!]$

HW ?'s: p. 69

33. $\lim_{x \to 5^+} \dfrac{x - 5}{x^2 - 25}$

35. $\lim_{x \to 2^+} \dfrac{x}{\sqrt{x^2 - 4}}$

Can also check table



37. $\lim_{\Delta x \to 0^+} \dfrac{2(x + \Delta x) - 2x}{\Delta x}$

$\lim_{\Delta x \to 0^+} \dfrac{2x + 2\Delta x - 2x}{\Delta x}$

$\lim_{\Delta x \to 0^+} \dfrac{2\cancel{\Delta x}}{\cancel{\Delta x}}$

$= 2$

39. $\lim\limits_{\Delta x \to 0^+} \dfrac{\dfrac{1}{x+\Delta x} - \dfrac{1}{x}}{\Delta x}$

$$\lim_{\Delta x \to 0^+} \frac{1}{\Delta x}\left[\frac{x}{x}\cdot\frac{1}{x+\Delta x} - \frac{1}{x}\frac{(x+\Delta x)}{(x+\Delta x)}\right]$$

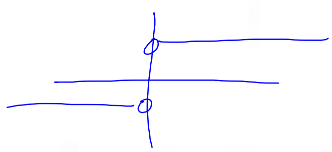
$$\lim_{\Delta x \to 0^+} \frac{1}{\Delta x}\left[\frac{x - x - \Delta x}{x(x+\Delta x)}\right]$$

$$\lim_{\Delta x \to 0^+} \left[-\frac{1}{x(x+\Delta x)}\right]$$

$$-\frac{1}{x^2}$$

41. $\lim\limits_{x \to 0} \dfrac{|x|}{x}$

43. $\lim\limits_{x \to 3} f(x),\quad f(x) = \begin{cases} \dfrac{x+2}{2}, & x \le 3 \\ \dfrac{12-2x}{3}, & x > 3 \end{cases}$



45. $\lim\limits_{x \to 1} f(x),\quad f(x) = \begin{cases} x^3+1, & x < 1 \\ x+1, & x \ge 1 \end{cases}$

$\lim\limits_{x \to 1^-} (x^3+1) = 2$

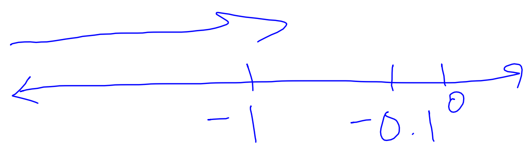
$\lim\limits_{x \to 1^+} (x+1) = 2$

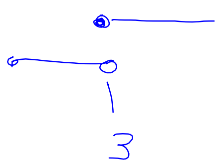
Since they match: $\lim\limits_{x \to 1} f(x) = 2$

47. $\lim_{x \to 3^-} 2[\![x - 3]\!]$

$= -2$

x	2.9	2.99
	$2[\![2.9-3]\!]$	$2[\![2.99-3]\!]$
	$2[\![-0.1]\!]$	$2[\![-0.01]\!]$
	$2(-1)$	$2(-1)$
	-2	-2





2.1 - 2.2: Strategies to find limits

1. Look at a graph
2. Look at a table
3. Direct substitution if the function is continuous
4. Factor and cancel to create a new function
5. Rationalize the numerator or denominator
6. Find a common denominator to simplify compound fractions

* Tues HW Quiz

Classwork Questions:

7) $f(x) = \sqrt[3]{1-x} + 2$

$f(x) = \sqrt[3]{-(x-1)} + 2$

$f(x) = \sqrt[3]{-1}\left(\sqrt[3]{x-1}\right) + 2$

$f(x) = -\sqrt[3]{x-1} + 2$

r_x Rt 1 up 2

8) $f(x) = -\ln(x+2)$

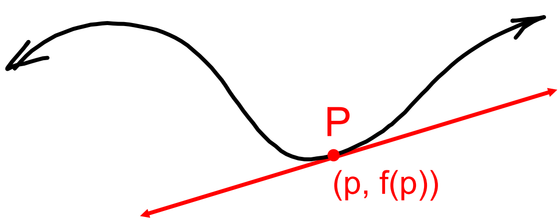
* check a point if not sure!

$0 \stackrel{?}{=} -\ln(-1+2)$

$0 = -\ln 1$

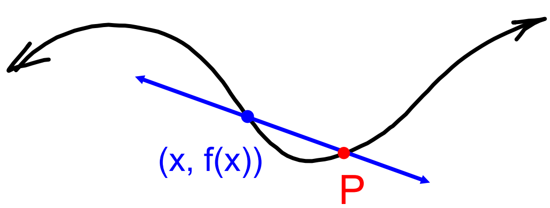
$0 = 0$ ✓

Finding the equation of a line tangent to a curve.



Point - Slope Form

$y - f(p) = m(x - p)$



$$m = \lim_{x \to p} \frac{f(x) - f(p)}{x - p}$$

You try:

$f(x) = x^2 + 3x + 4$ $\qquad$ $g(x) = 3 - x^2$

1. Find the point-slope equation of the line tangent to f(x) through (-3, 4).

2. Find the line tangent to g(x) where x = -2.

2.3 Continuity

-Determine continuity at a point

-Determine continuity on an open interval

-Determine continuity on a closed interval

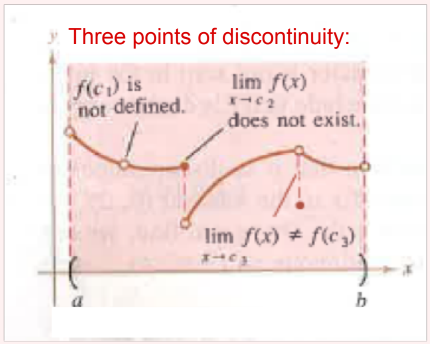
-Investigate properties of continuity

-Introduce the Intermediate Value Theorem

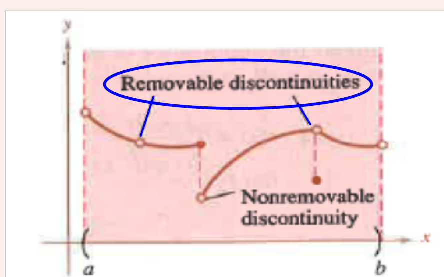
Definition of Continuity

Continuity at a point: A function f is called continuous at c if the following 3 conditions are met

1. $f(c)$ is defined (it has a value, a point on the graph)
2. $\lim_{x \to c} f(x)$ exists
3. $\lim_{x \to c} f(x) = f(c)$



A discontinuity is considered removable if by redefining that point it becomes continuous



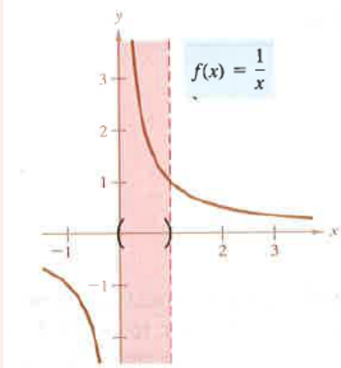
(a,b) means

"open interval from a to b"

Continuity on an <u>Open</u> Interval:
A function is called continuous on an open interval (a, b) if it is continuous at each point in the interval.

Decide whether the function is continous on the given interval:

(a) $f(x) = \frac{1}{x}, \quad (0, 1)$



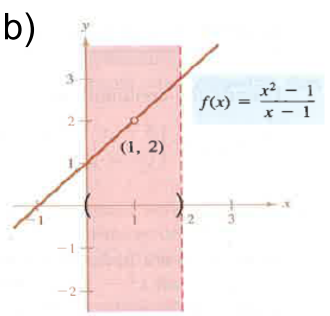
Continuous on (0, 1)

Continuity on an <u>Open</u> Interval:
A function is called continuous on an open interval (a, b) if it is continuous at each point in the interval.

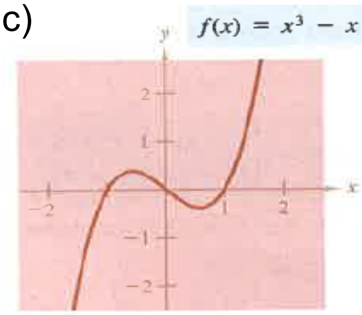
(b) $f(x) = \frac{x^2 - 1}{x - 1}, \quad (0, 2)$

(c) $f(x) = x^3 - x, \quad (-\infty, \infty)$

b)



c)

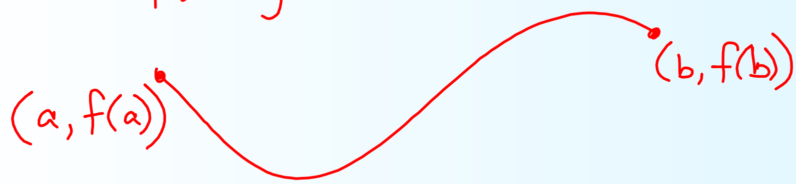


Continuity on a Closed Interval:
A function is continuous on a closed interval [a, b] if it is continuous on the open interval (a, b) and

$$\lim_{x \to a^+} f(x) = f(a) \quad \text{and} \quad \lim_{x \to b^-} f(x) = f(b).$$

approach a from right

approach b from the left



Ex: let b = 2

f(x) = 3x + 4

g(x) = x^2

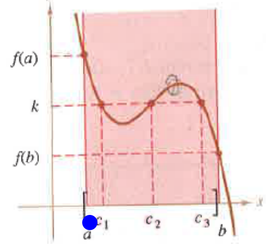
$f(g(c))$

Properties of Continuity: If b is a real number and f and g are continuous at $x = c$, then the following functions are also continuous at c.

1. Scalar multiple bf
2. Sum and difference $f \pm g$
3. Product fg
4. Quotient f/g if $g(c) \neq 0$

Continuity of composite functions: If g is continuous at c and f is continuous at $g(c)$, then the composite function given by $f(g(c))$ is continuous at c

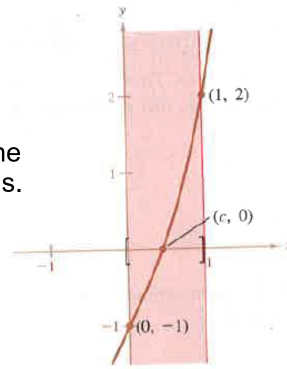
Intermediate Value Theorem: If f is continuous on $[a, b]$ and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a, b]$ such that $f(c) = k$



f is continuous.
(For k, there exist 3 c's.)

Use the Intermediate Value Theorem to show that *f(x)* has a zero on the interval [0, 1].

$f(x) = x^3 + 2x - 1$



f(x) is continuous on [0, 1] and since the outcomes for f(0) and f(1) change signs. we know f(x) crosses the x-axis.

HW: p. 75 (starts there)

1 - 45 eoo and 47

Get organized:

* All your HW from this week complete, labeled and in order for Tuesday's HW Quiz.

* Grapher

* Pencils, graph paper, binder or folder for handouts.